**Square Root Evaluation**

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**1 Introduction**

In mathematics, a square root of a number is a number such that . It is denoted as .

For example:

Now, it’s a point to notice that, for , .

This can be justified as follows:

We know,

and

and

So,

Now, it’s easy to find square roots of numbers that are perfect squares (those numbers which can be represented as product of two same numbers, like 9, 25, 36, 49, etc.).

But, it’s comparatively difficult to find square roots of non – perfect squares like 15, 8, etc.

**2 Square Root Evaluation for non – perfect squares**

Suppose is a function in (a mathematical tool that takes an input in and gives a result after processing).

Now, for a slight change, in the input argument of the function ,

Now, for Square Root, the function

and

So,

Illustration 1: Evaluate

Solution:

We know, , So, and

Now, we know,

Illustration 2: Evaluate

Solution:

We know, , So, and

Now, we know,

Illustration 3: Evaluate

Solution:

We know, , So, and

Now, we know,

**3 Square Root Evaluation using Babylonian Method**

The method that we are going to study in this section was initiated back in Babylonia.

The Algorithm for this method is quite simple, but quite long in big runs.

The steps are as follows:

* Let the number, whose square root is to found is .
* Start with two number, say, and .
* Set
* Set
* Modify as
* Modify as
* Check for

If , where is the accuracy fraction like , then REPEAT the steps 5, 6, 7 again.

Else, STOP.

Now,

Illustration 4: Evaluate with desired accuracy

Solution:

Let

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  | Is ? |
| 4 | 1 | 3 |  | YES |
|  |  | 0.9 |  | YES |
|  |  | 0.1 |  | YES |
|  | 2 | 0 |  | NO |

Now,